Compressed Sensing With Dynamical Neural Networks

Michael Teti, William Hahn, and Elan Barenholtz
Machine Perception and Cognitive Robotics Laboratory
Center for Complex Systems and Brain Sciences

Quick Look

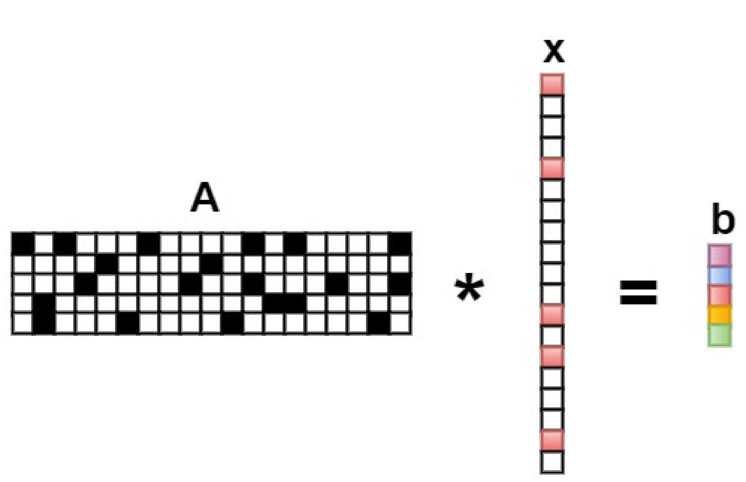
Biologically-motivated neural network that recovers compressively-sensed signals **over 8 times faster** than current state-of-the-art methods.

Introduction

Modern cameras/devices are wasteful of data, which can be expensive to collect and transmit. It doesn't make sense to collect much more data than you need, then compress it. Compressive sensing (CS) combines sampling and compression in one step.

An image x with n pixels can be transformed into a $n \times 1$ column vector, where all the pixels are stacked on top of each other.

To take compressed samples of x, multiply it with a random $m \times n$ matrix A, where $m << n$ to get compressed measurement vector b.



Get x back from b with the expression

$$\min_{\mathbf{x}} \ ||\mathbf{x}||_1 + \frac{1}{2}||\mathbf{A}\mathbf{x} - \mathbf{b}||_2^2 \qquad (1)$$

[1], where we add the sum of the reconstructed x to the minimization problem to induce sparsity, as there are many possible solutions and the correct one is the sparsest solution [2]. Current methods require hundreds of lines of code to solve this problem. If only there was a more simple, faster way . . .

Locally-Competitive Algorithms

Locally-Competitive Algorithms (LCAs) are a type of dynamical neural network that can be used to recover compressed signals using lateral inhibition like the human visual system [3]. Upon receiving input, each node charges up in proportion to how much the input resembles the appropriate stimulus. If it charges up enough, it will "fire" and produce an output, as well as inhibit nearby, similar nodes (red arrows in Fig. 1) in proportion to its activation.

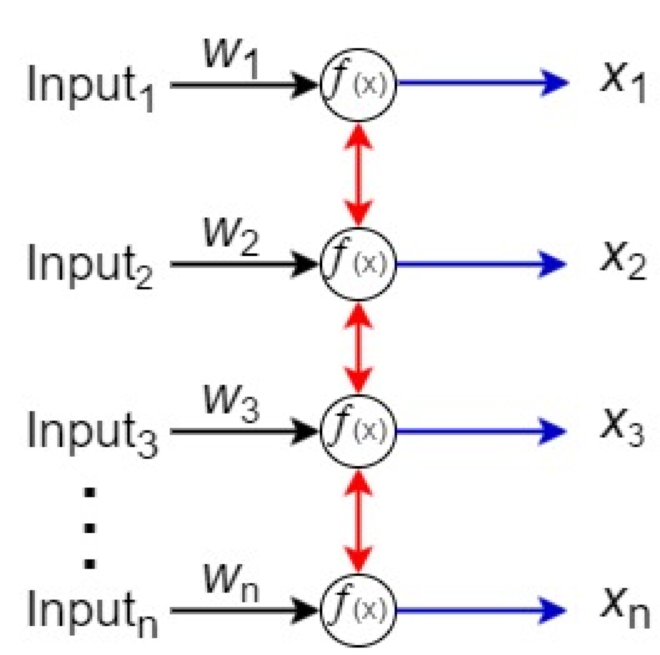


Figure 1: The network's weights, $w_1, w_2, ...w_n$ evolve over time to minimize the mean-squared error between $\mathbf{Ax}$ and $\mathbf{b}$.

Algorithm

lambda = 4.0
$h = 0.005$
u = zeros(n, 1)
While MSE **is above some value:**
$u = u + h * (A' * (b - A * x) - u - x)$
x=(u-sign(u).*(lambda)).*(abs(u)>(lambda))

As can be seen, the algorithm is extremely simple and efficient. Furthermore, it is a vectorized implementation, which enables the use of GPUs, making it much faster than alternative methods.

The first line receives the input and minimizes the mean-squared error between $\mathbf{Ax}$ and $\mathbf{b}$, while also causing the inhibition of nearby nodes.

The second line is the activation function of each node, which causes nodes with activations below threshold to output nothing. In all, the network outputs a sparse solution that approximates, or sometimes equals, the original image.

Compressive Sensing and Recovery of a Natural Image

1. We first turn the image into a single column vector with 12,371 elements
2. 6,185 compressed measurements are taken from the 12,371 pixel image by multiplying it by a random $6,185 \times 12,371$ matrix.
3. Next, the image, the random sensing matrix, and the 6,185 element measurement vector are sent to the LCA.
4. A discrete cosine transform (DCT) dictionary is added to the network, too, as this is required for natural images which aren't inherently sparse.
5. The LCA outputs a solution, which is then multiplied by the DCT dictionary again to approximately recover the image.

The DCT is used in the recovery process of natural images because it makes them sparse, and the correct solution is the sparsest one.

The LCA is able to approximately recover the image with only 50% of the measurements! It outputs a solution that is equivalent to the solution returned by the primal-dual interior-point method, the current state-of-the-art recovery algorithm.

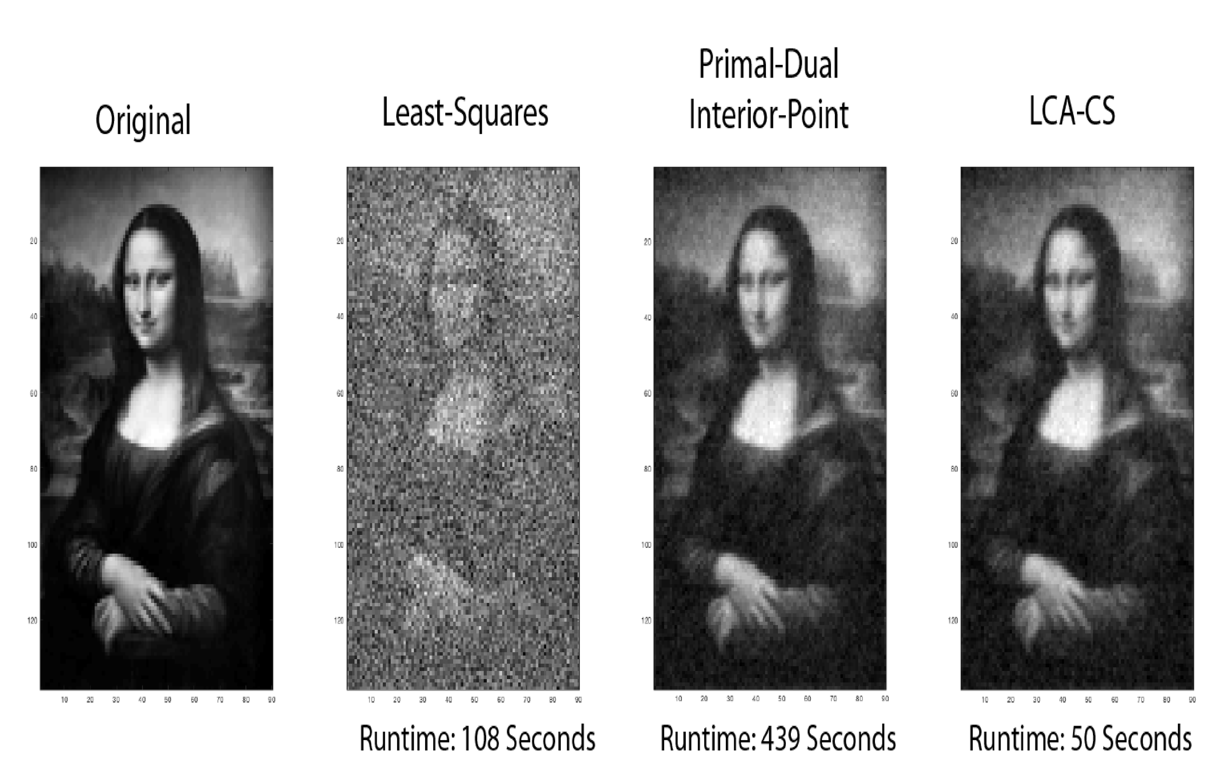
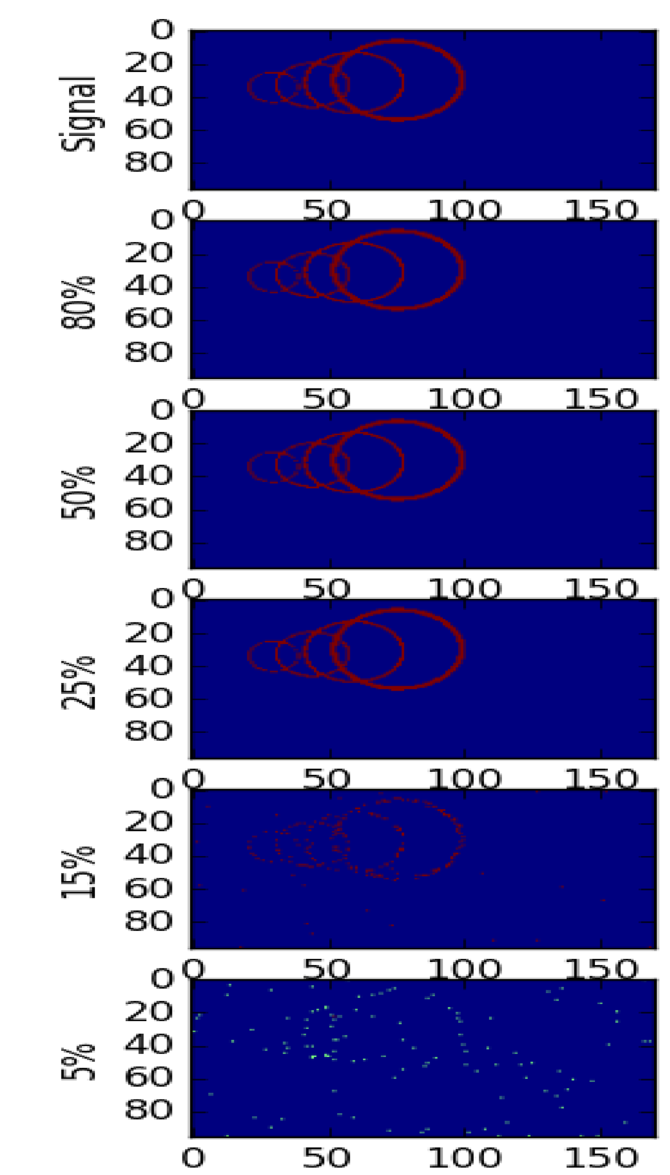


Figure 2: The original image and three reconstruction methods, the last of which is presented here.

How Low Can You Go?

Here, we explore the effect of different sampling rates on the reconstruction of an image.



Conclusion and References

The vast amount of data generated each day is already too much to store in hardware alone, and it is still growing. Compressive sensing offers a solution to this problem by taking fewer measurements that contain all of the information. Until now, the recovery process was slow, complicated, and not neurally-relevant. A simple locally-competitive neural network model is able to reconstruct signals from a fraction of the data and in a fraction of the time relative to other methods.

[1] William Edward Hahn, Stephanie Lewkowitz, Daniel C Lacombe, and Elan Barenholtz. Deep learning human actions from video via sparse filtering and locally competitive algorithms. *Multimedia Tools and Applications*, 74(22):10097–10110, 2015.

[2] Emmanuel J Candès, Justin Romberg, and Terence Tao. Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information. *IEEE Transactions on information theory*, 52(2):489–509, 2006.

[3] C.J. Rozell, D.H. Johnson, R.G. Baraniuk, B.A. Olshausen, and R.L. Ortman. Analog system for computing sparse codes, 2010. US Patent 7,783,459.

Contact Information

- Web: http://mtetiresearch.com
- Github: https://github.com/MichaelTeti
- Email: mteti@fau.edu
- MPCR Website: http://mpcrlab.com